11

الصف الأول ثانوي

رياضيات
(علمي)

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المتطابقات المثلثية

السؤال الأول: بسط العبارات المثلثية التالية :

1) $\dfrac{1-\cos^2\theta}{\cos^2\theta}$

2) $\dfrac{\cos\theta \cdot \csc\theta}{\cot\theta}$

3) $\dfrac{\cos^2\theta}{1-\sin\theta}$

4) $\dfrac{\sec^2\theta-1}{\sec^2\theta}$

5) $\dfrac{\tan\theta \cot\theta}{\tan\theta}$

السؤال الثاني : أثبت صحة المتطابقات التالية :

1) $\sin \theta \cdot \cot \theta \cdot \sec \theta = 1$

2) $\csc^2 \theta - \cos^2 \theta \csc^2 \theta = 1$

3) $(\sin \theta - \cos \theta)(\sin \theta + \cos \theta) = 1 - 2\cos^2 \theta$

4) $\dfrac{\sec \theta - \cos \theta}{\sec \theta} = \sin^2 \theta$

5) $\dfrac{\cos^4 t - \sin^4 t}{\cos^2 t} = 1 - \tan^2 t$

6) $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\csc A$

7) $1 - \cos 2x = \tan x \sin 2x$

8) $\cos^4 \theta - \sin^4 \theta = \cos 2\theta$

السؤال الثالث : دون استعمال الالة الحاسبة أوجد قيمة :

$\sin 105^{\circ}$ $\qquad\qquad\qquad$ $\cos 165^{\circ}$

السؤال الرابع : اذا كان $tan\,\theta = -\frac{3}{4}$ حيث $\frac{\pi}{2} < \theta < \pi$ فأوجد ما يلي :

1) $sin\,2\theta$

2) $cos\,2\theta$

3) $tan\,2\theta$

السؤال الخامس: اذا كان $sin\,\theta = \frac{5}{8}$ حيث $0 < \theta < 90$ فاوجد :

1) $sin\,\frac{\theta}{2}$

2) $cos\,\frac{\theta}{2}$

3) $tan\,\frac{\theta}{2}$

السؤال السادس : حل المعادلات المثلثية التالية حيث $0 \leq \theta < 360$ ،

1) $4\,cos^2\,x + 7\,sin\,x - 7 = 0$

2) $sin(x - 20) = \frac{1}{\sqrt{2}}$

3) $cos\,3x = -\frac{1}{2}$

السؤال الأول: بسط العبارات المثلثية التالية :

1) $\frac{1-\cos^2 \theta}{\cos^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$

2) $\frac{\cos \theta \cdot \csc \theta}{\cot \theta} = \frac{\cos \theta \cdot \frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}} = \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}} = \frac{\sin \theta \cdot \cos \theta}{\sin \theta \cos \theta} = 1$

3) $\frac{\cos^2 \theta}{1-\sin \theta} = \frac{1-\sin^2 \theta}{1-\sin \theta} = \frac{(1-\sin \theta)(1+\sin \theta)}{(1-\sin \theta)} = 1 + \sin \theta$

4) $\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \frac{\sec^2 \theta}{\sec^2 \theta} - \frac{1}{\sec^2 \theta} = 1 - \frac{1}{\frac{1}{\cos^2 \theta}} = 1 - \cos^2 \theta = \sin^2 \theta$

5) $\frac{\tan \theta \cot \theta}{\tan \theta} = \frac{\frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta}} = \frac{1}{\frac{\sin \theta}{\cos \theta}} = \frac{\cos \theta}{\sin \theta} = \cot \theta$

السؤال الثاني : أثبت صحة المتطابقات التالية :

1) $\sin \theta \cdot \cot \theta \cdot \sec \theta = 1$

$\sin \theta \cdot \cot \theta \cdot \sec \theta = \sin \theta \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} = \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} = 1$

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2) $csc^2 \theta - cos^2 \theta \, csc^2 \theta = 1$

$$csc^2 \theta (1 - cos^2 \theta) = csc^2 \theta (sin^2 \theta) = \frac{1}{sin^2 \theta} \cdot sin^2 \theta = 1$$

3) $(sin \theta - cos \theta)(sin \theta + cos \theta) = 1 - 2 cos^2 \theta$

$(sin \theta - cos \theta)(sin \theta + cos \theta)$

$= sin^2 \theta - sin \theta \cos \theta + sin \theta \cos \theta - cos^2 \theta$

$= sin^2 \theta - cos^2 \theta = (1 - cos^2 \theta) - cos^2 \theta = 1 - 2 cos^2 \theta$

4) $\dfrac{sec \theta - cos \theta}{sec \theta} = sin^2 \theta$

$$= \frac{\frac{1}{cos \theta} - cos \theta}{\frac{1}{cos \theta}} = \frac{\frac{1 - cos^2 \theta}{cos \theta}}{\frac{1}{cos \theta}} = \frac{\frac{sin^2 \theta}{cos \theta}}{\frac{1}{cos \theta}} = \frac{sin^2 \theta \cdot cos \theta}{cos \theta} = sin^2 \theta$$

5) $\dfrac{cos^4 t - sin^4 t}{cos^2 t} = 1 - tan^2 t$

$$= \frac{(cos^2 t - sin^2 t)(cos^2 t + sin^2 t)}{cos^2 t} = \frac{(cos^2 t - sin^2 t)(1)}{cos^2 (t)}$$

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$$\frac{\cos^2 t}{\cos^2 t} - \frac{\sin^2 t}{\cos^2 t} = 1 - \tan^2 t$$

6) $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\csc A$

$$= \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)(\sin A)} = \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{(1+\cos A)(\sin A)}$$

$$= \frac{1+1+2\cos A}{(1+\cos A)(\sin A)} = \frac{2+2\cos A}{(1+\cos A)(\sin A)} = \frac{2(1+\cos A)}{(1+\cos A)(\sin A)} = \frac{2}{\sin A} = 2\csc A$$

7) $1 - \cos 2x = \tan x \sin 2x$

$LHS = 1 - \cos 2x = 1 - (1 - 2\sin^2 x) = 1 - 1 + 2\sin^2 x =$

$0 + 2\sin^2 x = 2\sin^2 x$

$RHS = \tan x \sin 2x = \frac{\sin x}{\cos x} \times 2\sin x \cos x = 2\sin^2 x = LHS$

8) $\cos^4 \theta - \sin^4 \theta = \cos 2\theta$

$$= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = (\cos 2\theta)(1) = \cos 2\theta$$

السؤال الثالث : دون استعمال الالة الحاسبة أوجد قيمة :

$$cos\ 165°, \quad sin\ 105°$$

$$cos\ 165 = cos\ \frac{330}{2} = -\sqrt{\frac{1 + cos\ 330}{2}} = -\sqrt{\frac{1 + cos\ 30}{2}}$$

$$-\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2 + \sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 + \sqrt{3}}{4}} = -\frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$sin\ 105° = sin\ \frac{210}{2} = +\sqrt{\frac{1 - cos\ 210}{2}} = +\sqrt{\frac{1 + cos(30)}{2}}$$
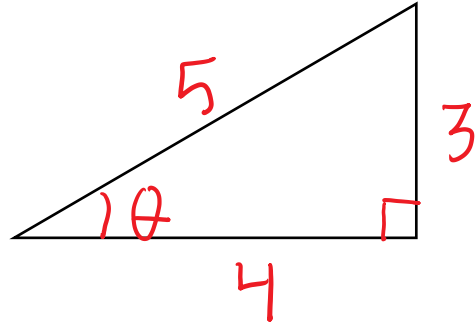
$$= \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

<u>**السؤال الرابع**</u> : اذا كان $tan\,\theta = -\frac{3}{4}$ حيث $\frac{\pi}{2} < \theta < \pi$ فأوجد ما يلي :

الحل :

$$sin\,\theta = \frac{3}{5}$$

$$cos\,\theta = -\frac{4}{5}$$

1) $sin\,2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{3}{5} \cdot \frac{-4}{5} = \frac{-24}{25}$

2) $cos\,2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \left(\frac{16}{25}\right) - \left(\frac{9}{25}\right) = \frac{7}{25}$

3) $tan\,2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{-3}{4}\right)}{1-\left(\frac{-3}{4}\right)^2} = \frac{-\frac{6}{4}}{1-\frac{9}{16}} = \frac{-\frac{6}{4}}{\frac{7}{16}} = -\frac{24}{7}$

<u>**السؤال الخامس**</u> :اذا كان $sin\,\theta = \frac{5}{8}$ حيث $0 < \theta < 90$ أوجد :

$$sin^2\,\theta + cos^2\,\theta = 1$$

$$\left(\frac{5}{8}\right)^2 + \cos^2\theta = 1$$

$$\frac{25}{64} + cos^2\,\theta = 1$$

$$cos^2\,\theta = 1 - \frac{25}{64} = \frac{39}{64} \rightarrow \cos\theta = \frac{\sqrt{39}}{8}$$

1) $sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}} = +\sqrt{\dfrac{1-\frac{\sqrt{39}}{8}}{2}} = \sqrt{\dfrac{8-\sqrt{39}}{16}} = \dfrac{\sqrt{8-\sqrt{39}}}{4}$

2) $cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}} = +\sqrt{\dfrac{1+\frac{\sqrt{39}}{8}}{2}} = \sqrt{\dfrac{8+\sqrt{39}}{16}} = \dfrac{\sqrt{8+\sqrt{39}}}{4}$

3) $tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\dfrac{1-\frac{\sqrt{39}}{8}}{1+\frac{\sqrt{39}}{8}}} = \sqrt{\dfrac{\frac{8-\sqrt{39}}{8}}{\frac{8+\sqrt{39}}{8}}} = \sqrt{\dfrac{8-\sqrt{39}}{8+\sqrt{39}}}$

السؤال السادس: حل المعادلات المثلثية التالية حيث $0 \le \theta < 360$:

1) $4\cos^2 x + 7\sin x - 7 = 0$

$4(1 - \sin^2 x) + 7\sin x - 7 = 0$

$4 - 4\sin^2 x + 7\sin x - 7 = 0$

$-4\sin^2 x + 7\sin x - 3 = 0$

$4\sin^2 x - 7\sin x + 3 = 0$

$(4\sin x - 3)(\sin x - 1) = 0$

$$4 \sin x - 3 = 0$$

$$4 \sin x = 3$$

$$\sin x = \frac{3}{4}$$

$$x = \sin^{-1} \frac{3}{4} = 48.6°$$

$$x = 180 - 48.6 = 131.6°$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

$$x = \sin^{-1}(1) = 90^0 = \frac{\pi}{2}$$

2) $\sin(x - 20) = \frac{1}{\sqrt{2}}$

$$x - 20 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45° \rightarrow x = 65°$$

$$x - 20 = 135° \rightarrow x = 155°$$
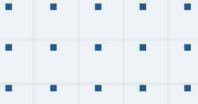
3) $\cos 3x = -\frac{1}{2}$

$$3x = \cos^{-1}\left(-\frac{1}{2}\right) = 120°$$

$$3x = 120 + 2\pi k \quad , \quad k \in Z$$

$$x = 40 + \frac{2\pi k}{3}$$

$$x = 40° , 80° , 160° , 200° , 280° , 320°$$